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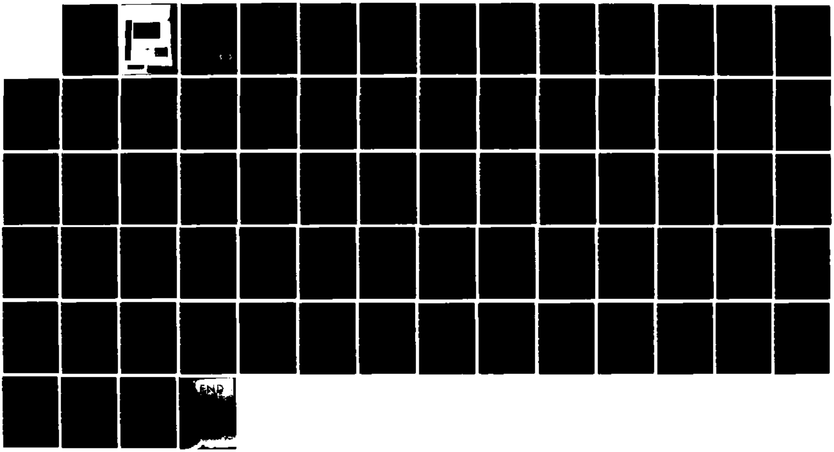

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A THEORY OF PREFERENCE REVERSALS

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August 1984

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The preference reversal phenomenon refers to the fact that people who choose gamble A over B often ask for more money to sell B than A. This finding is remarkably robust over subjects, experiments, differing incentives, and types of gambles (e.g., gains vs. losses). However, previous research has confounded response method (judgment vs. choice) with the "worth" scale

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on which the response is expressed (prices in dollars vs. attractiveness of the gamble). When these two factors are crossed in a 2 x 2 design, 6 pairs of preference reversals are theoretically possible. An experiment to test for the existence of these reversals revealed that 5 out of the 6 types were significant. A theory to explain these results was developed in which the basic evaluation of a gamble, assumed to be a function of the utilities and probabilities of the payoffs, is translated onto various worth scales via a subjective interpolation process. This process involves the matching of proportional adjustments on the utility scale to those on the worth scales (prices, ratings, etc.). The model accounts for the direction of all 5 reversals and correctly predicts that some directions are impossible. The model is tested on new data from a study by Tversky and Slovic (1984) and fits their data well. In addition, the model is shown to be consistent with reversals for gambles involving losses, violations of dominance (e.g., increasing the amount of a loss increases the attractiveness rating of the gamble), the prediction of numerical values of prices and ratings, and extensions to other assessment methods. We also compare our theory to a differential weighting model proposed by Tversky and Slovic (1984).

A THEORY OF PREFERENCE REVERSALS

The evaluation of gambles is a ubiquitous cognitive activity that underlies perception (Green & Swets, 1966), judgment (Kahneman, Slovic, & Tversky, 1982), and choice (Wallsten, 1980). It is therefore an important matter to understand how people evaluate and choose between alternatives in the face of uncertainty. Indeed, the psychological literature on risk has continued to grow and influence other disciplines as well (e.g., economics). However, much psychological theorizing about decision-making-under-uncertainty has been heavily influenced by economic theories of how "rational" actors are supposed to behave in order to maximize their own welfare. While this has been a useful starting point, rational actors seem less prone to attentional shifts, memory lapses, information overload, and other cognitive limitations, than the rest of us. In order to develop useful and realistic descriptive theories, one strategy has been to focus on discrepancies in behavior between idealized rational actors and real people. In this way, one attempts to find psychological factors and processes that will explain the discrepancies and predict new phenomena. In this paper, we consider one of the most interesting discrepancies, the so-called "preference reversal" phenomenon, first investigated by Lichtenstein and Slovic (1971) and subsequently replicated by many others (Grether & Plott, 1979; Hamm, 1979; Lichtenstein & Slovic, 1973; Lindman, 1971; Mowen & Gentry, 1980; Pommerehne, Schneider, & Zweifel, 1982; Reilly, 1982). A review of these findings is given by Slovic and Lichtenstein (1983).

The phenomenon

Consider the following two gambles, one of which has a high probability of winning a small amount of money (called the P-Bet) and the other, which has a low probability of winning a large amount of money (called the $-Bet):

P-Bet: Win \$4 with $p = .97$

Lose \$1 with $p = .03$

\$-Bet: Win \$16 with $p = .31$

Lose \$1.5 with $p = .69$

When subjects are asked to choose which gamble they prefer to play, most choose the P-Bet over the \$-Bet. However, when each gamble is presented singly and subjects are asked to state their minimum selling prices for corresponding lottery tickets, the \$-Bet receives a higher price than the P-Bet. If it is assumed that higher selling prices also reflect preferences, then the order of preference reverses depending on whether one chooses, or states selling prices.

Preference refersals have not only been found in laboratory studies (some with elaborate controls--see Grether & Plott, 1979), they have also been demonstrated at the Four Queens Casino in Las Vegas (Lichtenstein & Slovic, 1973). Therefore, the phenomenon is robust over subjects, experimenters, various experimental conditions, as well as the amount of incentives to "do better."

Importance of preference reversals

In addition to its obvious practical importance, preference reversals pose a theoretical challenge to almost all theories of judgment and choice. To see why, suppose that a person sets a minimum selling price of \$a for commodity A, and \$b for commodity B, where \$a > \$b. Given a choice between commodity A and an amount of money \$c, where \$a > \$c > \$b, the person should choose A. To choose \$c over A would be, in effect, to sell A for \$c despite the fact that \$c is less than A's minimum selling price. Similarly, given a choice between \$c and B, the person should choose \$c since it is larger than B's minimum selling price. Since A is chosen over \$c, and \$c is chosen over B, transitivity of choice requires that A be chosen over B. However, the preference reversal phenomenon demonstrates that although \$a > \$b, people often choose B over A.

Thus, the phenomenon implies that either some very simple assumptions about selling prices are wrong, or choice is consistently intransitive. In any event, the theoretical challenge is to understand how such behavior can happen, and to explicate the conditions for both its occurrence and non-occurrence.

Possible explanations

The prevailing view of the preference reversal phenomenon, due to Lichtenstein and Slovic (1971, 1973), is that, "variations in response mode cause fundamental changes in the way people process information, and thus alter the resulting decisions" (Lichtenstein & Slovic, 1973, p. 16). Grether and Plott (1979), who considered no fewer than thirteen possible economic, psychological, and artifactual explanations for the phenomenon, were unable to reject the notion that the type of information processing subjects perform is conditional on the response mode. Lichtenstein and Slovic suggest a more specific hypothesis; viz., that subjects set their minimum selling prices by a process of anchoring-and-adjustment. That is, subjects who find a gamble attractive are thought to anchor on the amount they stand to win, and then adjust downward for the amount and probability to lose. However, the adjustment is thought to be typically insufficient (cf. Tversky & Kahneman, 1974), leading to prices that are inconsistent with choices, which are presumably achieved in some other way.

More recently, Tversky and Slovic (1984) have suggested a compatibility hypothesis, which states that, ". . . the easier the mapping of a stimulus component onto the response scale, the greater the weight attached to this component" (p. 6). In particular, because a gamble's payoffs and its minimum selling price are both expressed in monetary units, this compatibility is thought to result in a greater weighting of payoffs in setting prices than in choices or other non-monetary responses. Later in this paper, we reanalyze

Tversky and Slovic's (1984) data from a different perspective and discuss the compatibility hypothesis at greater length.

Although psychologists and economists have both been interested in the preference reversal phenomenon, the two disciplines have tended to interpret it differently. In particular, psychologists have tended to view reversals as a discrepancy between judgment and choice. For example, Fischhoff, Slovic, and Lichtenstein (1980, p. 128) say, "people use different cognitive processes when evaluating the worth of gambles via a comparative model ('Which would you rather play?') than they use when judging each gamble separately ('How much is playing each worth to you?')." However, economists have tended to give the phenomenon a subtly different interpretation, one that leaves the compatibility of judgment and choice an open question. For example, Grether and Plott (1979, p. 623) see reversals as being inconsistent with the "basic theoretical proposition" that "an individual should place a higher reservation price on the object he prefers." In this view, it is irrelevant that the preference order is elicited via choices, while the ordering of prices is obtained via judgments. The critical discrepancy is between price and preference, not between judgment and choice.

Distinguishing between the two interpretations hinges on a distinction between, on the one hand, what the subject has been asked to <u>do</u>, i.e., judge or choose, and on the other hand, what <u>scale</u> the subject has been asked to do it with, i.e., minimum selling price or attractiveness. We will refer to judgment and choice as two "response methods" with which to assess subjective worth. Further, just as subjective worth can be assessed with different response methods, it can also be assessed with what we will call different "worth scales," including minimum selling price and attractiveness. Other worth scales include desirability, maximum buying price, etc. All these variables putatively

reflect (at least monotonically) the same underlying subjective worth. Moreover, each worth scale can be combined with any of several response methods.

The preference reversal phenomenon constitutes an incompatibility between one kind of judgment and one kind of choice. However, the response method is confounded with the worth scale. As indicated by the arrow in Figure 1, the usual reversal involves the observation of an incompatibility between judgments made with one worth scale--minimum selling price--and choices made with a different worth scale--attractiveness. Therefore, one can view preference

Insert Figure 1 about here

reversals as representing either an incompatibility between the worth scales of minimum selling price and attractiveness, or, an inconsistency between the response methods of judgment and choice. Since previous studies have confounded response method and worth scale, the interpretation is unclear.

It is important to note that response method does not have to be confounded with worth scale. As shown in Figure 1, it is possible to construct tasks so that minimum selling price and attractiveness can each be assessed by judgment or choice. Thus, in addition to the usual tasks, (Grether & Plott, 1979; Lichtenstein & Slovic, 1971; 1973; Mowen & Gentry, 1980; Pommerehene *et al*., 1982; Reilly, 1982),[1] it is also possible to assess the ordering of minimum selling prices with the response method of choice (lower left cell).[2] To do so, the subject is asked to indicate the gamble for which he or she would hold out for the higher price. (Note that the subject is not asked to indicate the gamble for which he or she would "ask" the higher price, for this might encourage strategic behavior and produce something other than a choice with respect to *minimum* selling price.) The upper right cell shows how one can assess attractiveness with the judgment response method. This cell bears some elaboration, for it illustrates why the term "attractiveness" is being used

WORTH SCALE

RESPONSE METHOD		Minimum Selling Price	Attractiveness
	Judgment	What is the smallest price for which you would sell this gamble?	How much would you like to play this gamble, on a scale from 1 to 100?
	Choice	For which of these two gambles would you hold out for the higher price?	Which of these two gambles would you prefer to play?

Figure 1. Combinations of response method and worth scale.

rather than "preference." There is a tendency to think that "preference" is naturally confounded with choice; one speaks of having preferences "between" options. On this view, the task described in the upper right cell assesses something other than "preference." However, the idea that choices are determined by the relative magnitudes of two levels of an attribute goes back to Fechner (1860) and Thurstone (1927), who referred to these magnitudes as "discriminal processes." If we adopt this view, it seems reasonable to use the judgment response method to measure the magnitudes that are thought to underlie choice.

This paper has two goals. The first is to clarify the nature of the preference reversal phenomenon and determine exactly what it is that requires explanation. The experiment to follow removes the confounding of response method and worth scale, and is designed to determine which, if either, of the two proposed interpretations of the phenomenon can be rejected. The second goal is to develop and test a theory for the phenomenon as it is revealed in the experimental results.

The remainder of this paper is organized as follows. The experiment is reported in the next section. To anticipate the findings, the results show that the discrepancy between judgment and choice contributes to, but does not completely account for, the preference reversal phenomenon. In the following section, we turn to a theoretical account of the results. Implications of this theory are then tested by reanalyzing an experiment by Tversky and Slovic (1984). Thereafter, we discuss various implications and extensions of our model.

PREFERENCE REVERSAL EXPERIMENT

Method

<u>Subjects</u>. The subjects were 11 male and 12 female student volunteers from

the University of Michigan paid subject pool.

<u>Stimuli</u>. The 12 two-outcome gambles used as stimuli are shown in Table 1. These gambles have been used in several previous studies, beginning with the experiment reported by Lichtenstein and Slovic (1971). The 12 gambles were

Insert Table 1 about here

arranged in 6 pairs, each pair containing a P-Bet and a $-Bet. Within each pair, the two gambles have approximately the same expected value. Each stimulus was displayed as a wheel of fortune on a 4" × 6" card. The winning and losing amounts were displayed numerically while probabilities were displayed both numerically and by pie sections on the wheel of fortune.

<u>Procedure</u>. Each subject was run individually in an experimental session that lasted no longer than one hour. Subjects responded to the 12 gambles four times, once for each combination of response method and worth scale. The order in which subjects performed these four tasks was varied, within the constraint that subjects alternated between the two worth scales. The order of the 12 gambles was randomized for each judgment trial and the order of the six pairs was randomized for each choice trial.

Subjects were asked to make judgments and choices using both worth scales. In the judgment-minimum selling price situation, subjects simply stated the lowest price they would require to sell each gamble. If a subject named a minimum selling price that was larger than the amount to win or a lower price than the (negative) amount to lose, it was taken as an indication that the subject did not understand the task. (A negative selling price corresponds to paying someone else to play the gamble, i.e., the subject suffers a sure loss.) The task was then re-explained and the subject could change any minimum selling price. No subject changed a minimum selling price other than the one which prompted the re-explanation. For the judgment-

TABLE 1

Stimuli for Experiment

Pair	Type	Probability of winning	Amount to win	Amount to lose	Expected Value
1	p	35/36	$ 4.00	$1.00	$3.86
	$	11/36	$16.00	$1.50	$3.85
2	p	29/36	$ 2.00	$1.00	$1.42
	$	7/36	$ 9.00	$ .50	$1.35
3	p	34/36	$ 3.00	$2.00	$2.72
	$	18/36	$ 6.50	$1.00	$2.75
4	p	32/36	$ 4.00	$ .50	$3.50
	$	4/36	$40.00	$1.00	$3.56
5	p	34/36	$ 2.50	$ .50	$2.33
	$	14/36	$ 8.50	$1.50	$2.39
6	p	33/36	$ 2.00	$2.00	$1.67
	$	18/36	$ 5.00	$1.50	$1.75

attractiveness situation, subjects were asked to rate the degree to which they would like to play the gamble on a 100-point scale. In the choice-minimum selling price condition, subjects were asked to indicate the gamble for which they would demand the higher price. In the choice-attractiveness case, subjects were asked to choose the gamble they would most like to play.

Data analysis. Each subject made 24 judgments (12 minimum selling prices and 12 ratings of attractiveness) and 12 choices (6 using minimum selling prices and 6 using attractiveness). If we simply consider the ordering of judgments and the ordering on choices, one can compare any pair of conditions as to reversals. However, while the six possible pairs of orderings are not independent, all six bear on the issue of reversals and were thus examined. In order to illustrate our procedure, consider the judgments of minimum selling price and the choices based on attractiveness. For each of the 6 choices, we examined the minimum selling prices to see if their order was consistent with that of the choices. In this way, one can check for reversals in either of two directions: (1) the P-Bet is chosen over the $-Bet, but the $-Bet is given a higher minimum selling price; or, (2) the $-Bet is chosen over the P-Bet, but the P-Bet is given a higher minimum selling price. The same logic holds for comparing the two types of judgments, the two types of choices, and the three other comparisons between judgments and choices.

As in previous investigations, we tested for the presence of preference reversals by comparing the relative frequencies of the two types of reversals. The rationale for this procedure is as follows: If we assume that inconsistent orderings are due solely to random errors on the part of the subjects, there are two kinds of mistakes they can make; viz., falsely favoring the P-Bet, and, falsely favoring the $-Bet. If the probability of these two errors is unaffected by the task in which the ordering is assessed, reversals

should occur equally often in either direction. (For a somewhat different argument leading to the same statistical test, see Lichtenstein and Slovic, 1973.) Now consider ties. Suppose a subject chooses a P-Bet but gives equal ratings to the P and $ bets. Strictly speaking, the two orderings are discrepant and a kind of reversal has occurred. However, we take a conservative approach to counting reversals and ignore these cases. Since ties occurred infrequently in our data, the substantive results do not change in any way.

In order to test for the difference in the proportions of the two types of reversals, McNemar's test for the equality of correlated proportions (1969) was considered. In our case, however, each of the 23 subjects provided 6 responses, one for each pair of gambles. Pooling these responses across subjects violates the assumption of McNemar's test since the observations are not independent. Therefore, statistical tests were conducted using a method recently developed by Smith (1981).[3] Smith's method allows for particular dependencies in the data; specifically, conditional on the responses arising from different subjects, the response patterns are assumed to be independent. However, it is not assumed that different response patterns are unconditionally independent. Smith's test takes this lack of independence into account. The test yields an F-statistic along with its corresponding p-value.

Results

For each task, we can examine the number of responses that favored either the P-bet or the $-bet. The resulting 2 × 2 table for each of the 6 possible

Insert Table 2 about here

comparisons is shown in Table 2. Our first results concern the proportion of response patterns yielding strict reversals (excluding ties) in each of the 6 comparisons. These results are shown just below each 2 × 2 contingency

TABLE 2

Comparisons of Response Method/Worth Scale Combinations*

(1)

Choice/Attract. \ Judgment/Min.Sell.	P	$
P	26	40
$	13	48

53/138 = .38; F=8.2, p<.01

(2)

Choice/Attract. \ Judgment/Attract.	P	$
P	66	6
$	41	23

47/138=.34; F=17.1, p<.001

Choice/Min.Sell. \ Judgment/Min.Sell.	P	$
P	30	32
$	9	56

41/138=.30; F=7.2, p<.025

Choice/Min.Sell. \ Judgment/Attract.	P	$
P	56	9
$	51	20

60/138=.43; F=14.7, p<.001

Choice/Attract. \ Choice/Min.Sell.	P	$
P	46	27
$	20	45

47/138=.34; F=.54, n.s.

Judg./Min.Sell. \ Judgment/Attract.	P	$
P	36	3
$	65	22

68/138=.49; F=30.4, p<.001

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*Tied judgments not tabulated in contingency tables.

table. Each proportion is the sum of the off-diagonal cell entries divided by the total number of responses (6 responses/subject × 23 subjects = 138 responses).

Note that the proportion of reversals is substantial, ranging from .49, for the judgments across the two worth scales, to .30 for judgments vs. choices (using minimum selling prices). Over all conditions, the average proportion of reversals was .38. Although there were considerable reversals, do they represent systematic tendencies or do they simply reflect random error in the data? In order to test this, recall that we used Smith's test for comparing whether the proportions of both types of reversals were equal (i.e., we tested for equality of the off-diagonal cell entries). The existence of systematic reversals is indicated only if the reversal pattern is asymmetric, i.e., one type of reversal is more prevalent than the other. The results of these tests are also shown in Table 2. First, note that 5 of the 6 reversal patterns are significant, including the one found in previous research (shown in the upper left 2 × 2 table). Therefore, our results not only replicate the usual finding, they show the existence of several new kinds of reversals that are quite substantial. Second, consider the comparisons involving judgment vs. choice (the upper four 2 × 2 tables). All of these are significant, providing strong evidence for the contention that judgments and choices are not psychologically equivalent. Third, while judgment vs. choice is sufficent for reversals, it is not necessary. Note that the strongest reversal effect is for the comparison of minimum selling price and attractiveness judgments (shown in the lower right 2 × 2 table). On the other hand, when comparing choices over different worth scales, no systematic reversals were found. To sum up, our results indicate that both response method and worth scale are affecting reversals although the latter only occurs for judgments.

In addition to the overall magnitude of reversals, it is important to know their direction. We investigated this by examining each of the five significant reversal patterns. The upper two tables in column 1 show the pattern of results for judgments of minimum selling prices with both types of choices. In the first table, the typical reversal finding is shown; i.e., more choices of the P-Bet and a higher minimum selling price to the $-Bet than vice versa. Note that the same result obtains when the choice is with respect to minimum selling price. The three tables in column 2 show the results for the attractiveness judgments. The bottom table shows the comparison of the two types of judgments. Note that the reversal pattern is very strong. The two tables comparing attractiveness judgments with choices also show strong reversals of the same type; i.e., subjects chose the $-Bet but rated the P-Bet as more attractive to play. Overall, these results pose an important challenge for understanding the dynamics of preference reversals. That is, any theory of the phenomenon must come to grips with the fact that there are many types of reversals; and, the differences in the strength and direction of reversals varies according to the different combinations of response method and worth scale. We now present a theory of preference reversals and then return to discuss how it accords with the results given above.

THEORETICAL ANALYSIS: EXPRESSION THEORY

The theory offered here draws on a conception of the subject's task as consisting of three stages: (1) perceiving and encoding the stimuli; (2) processing the encoded information; and (3) expressing a response. This three-stage conceptualization is consistent with ideas to be found in functional measurement (for example, Anderson, 1970; Birnbaum, 1974, 1978; Birnbaum, Parducci, & Gifford, 1971; Birnbaum & Veit, 1974).

During the perceptual stage, the subject arrives at a psychological

representation of the gambles. This representation may take the form of encoding a subjective value for each dimension of the gamble (i.e., probabilities and outcomes are encoded as subjective probabilities and utilities); or, the subject may more or less directly perceive the gambles as having a particular amount of some psychologically relevant dimension, such as risk. During the processing stage, the subject integrates the previously encoded information to arrive at an overall impression or basic evaluation of the gambles. Finally, during the response stage, the subject expresses the basic evaluation in the form of a response requested by the experimenter.

One could plausibly argue that any or all of the three stages are affected by such factors as the framing of acts, contingencies, and outcomes (Slovic, Fischhoff, & Lichtenstein, 1982; Tversky & Kahneman, 1981), or by the response method and the worth scale. In our view, the simplest way to account for the effects revealed by our experiment is to focus on the response stage, particularly as it concerns how the basic evaluation is "translated" into a particular scale. To underscore that our emphasis is on the mapping of basic evaluations into responses, i.e., the expression of opinion rather than its formation, we refer to the model detailed below as Expression Theory.

Basic evaluation

We hypothesize that people arrive at their basic evaluations by a process of anchoring-and-adjustment; specifically, they anchor on how they would feel about receiving W outright, and then adjust downward to account for the probability of losing and the amount to be lost.

Formally, let $G = (W, p; L, 1-p)$ denote a gamble in which amount W is received with probability p and amount L is received with probability $1-p$, where $0 < p < 1$. It will be convenient to assume that $W > L$. We will call W and L the amounts to "win" and "lose," respectively, even though

both may be positive or negative. Let, $u(G)$ denote the basic evaluation of gamble G, which we will also somewhat loosely call the "utility" of G. We can then write,

$$u(G) = u(W) - \Delta[u(W) - u(L)] \qquad (1)$$

where $u(W)$ is the basic evaluation of the degenerate gamble in which the subject is certain to receive W, $u(L)$ is the basic evaluation of the degenerate gamble in which the subject is certain to receive L, and Δ is an adjustment weight between 0 and 1

The quantity $[u(W) - u(L)]$ represents the maximum reasonable adjustment. An adjustment in excess of $[u(W) - u(L)]$ would result in G receiving a less favorable evaluation than the sure receipt of L. The quantity Δ represents the proportion of this maximum reasonable adjustment which the subject feels is appropriate in the case of gamble G. A complete theory of basic evaluations would include a model for the adjustment proportion Δ. However, in accordance with our desire to focus on the <u>expression</u> of opinion instead of its formation, we leave the model of basic evaluation as general as possible, writing only,

$$\Delta = [u(W) - u(G)]/[u(W) - u(L)] \qquad (2)$$

This makes Equation (1) a simple identity. The importance of Δ is that it expresses the proportional adjustment in utility for the gamble due to uncertainty. In order to make Δ a meaningful quantity, we assume u to be an interval scale (Appendix A outlines conditions under which u is an interval scale).

Choice

The experiment did not reveal systematic reversals between choice when varying the worth scale. Therefore, we do not distinguish between the types

of choice in the theory. Furthermore, subjects always chose between pairs of simple two-outcome gambles with easily discriminable probabilities and outcomes. In these simple circumstances, we assume that each gamble (as opposed to each pair of gambles--see Tversky, 1969) receives a basic evaluation, and that subjects will always choose the gamble whose basic evaluation is more favorable. In more complicated circumstances, other considerations than the basic evaluation, such as the similarity of alternatives, may be considered in arriving at a choice (see, for example, Tversky, 1972).

Judgment

Minimum selling price. In order to set a minimum selling price for gamble G, MS(G), the subject must find some way to transform the basic evaluation, u(G), into a monetary amount. We hypothesize that subjects do this by a process of subjective interpolation whereby they try to equate the proportional adjustment in basic evaluation, Δ, with the proportional adjustment in the payoffs, $\Delta' = [W-MS(G)]/[W-L]$. We assume that subjects are "matching" their proportional adjustments so that Δ and Δ' are monotonically related. That is,

$$\begin{aligned} &\Delta_1 > \Delta_2 \quad \text{if and only if (abbreviated "iff")} \\ &\Delta_1' > \Delta_2' \end{aligned} \tag{3}$$

Equation (3) implies the existence of a strictly increasing function f such that, $\Delta' = f(\Delta)$. Therefore,

$$MS(G) = W - f(\Delta)[W - L] \tag{4}$$

where $\Delta = [u(W) - u(G)]/[u(W) - u(L)]$ as before.

Figure 2 shows a schematic drawing of the hypothesized process people

Insert Figure 2 about here

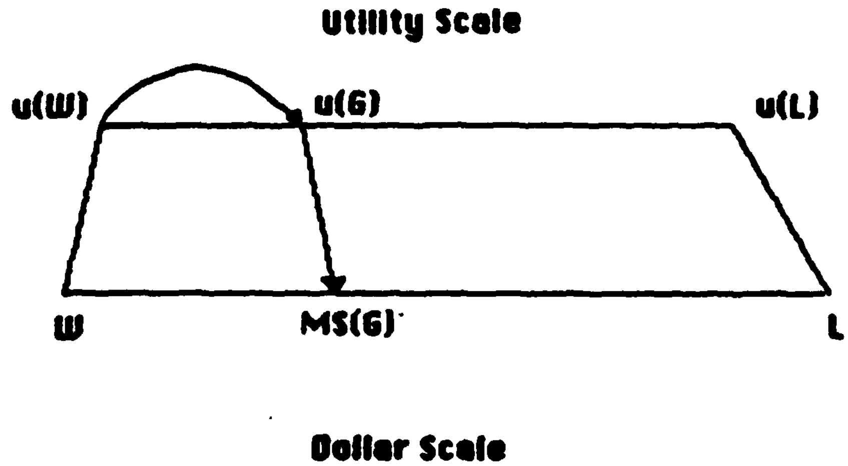

Figure 2. Translation of utility scale to dollar scale.

use to set their minimum selling prices. First, a process of anchoring and adjustment is used to reach a basic evaluation of gamble G on the utility scale. This basic evaluation is then translated onto the monetary scale by a process of subjective interpolation. That is, the person first establishes a correspondence between the endpoints of the utility and monetary scales (i.e., the amounts to win and lose). Then, a point on the monetary scale is sought such that the proportional adjustment in basic evaluation, Δ, matches the proportional adjustment in the payoffs, Δ'.

<u>Attractiveness</u>. A subject who must rate the attractiveness of gamble G. $R(G)$, is again faced with the problem of transforming $u(G)$ into a numerical judgment. Again, we hypothesize that subjects do this by a process of subjective interpolation, in which they try to equate the proportional adjustment in basic evaluation, Δ, with the proportional adjustment in the ratings, $\Delta'' = [100 - R(G)]/[100 - 1]$.[4] We assume that subjects are "matching" their proportional adjustments so that Δ and Δ'' are monotonically related. That is

$$\begin{array}{ll} \Delta_1 > \Delta_2 & \text{iff} \\ \Delta_1'' > \Delta_2'' & \end{array} \qquad (5)$$

Equation (5) implies the existence of a strictly increasing function g such that, $\Delta'' = g(\Delta)$. Therefore,

$$R(G) = 100 - g(\Delta)[100 - 1] \qquad (6)$$

Systematic reversals

In order to see how the theory presented above accounts for the reversals in our experiment, we can greatly simplify our presentation by assuming that the losses for both the P-Bet and \$-Bet are zero, $u(0) = 0$, and f and g are the identity function. Note that in the actual experiment, the losses

were generally small (see Table 1) and the difference in losses for the P and $-Bets was usually 50 cents. The simplifications introduced here do not greatly change the ordinal conclusions; however, we provide Appendix B for the general cases.

Given our assumptions above, we can re-write some of the basic equations as follows;

$$MS(G) = (1 - \Delta)W \tag{7}$$

where,

$$\Delta = [u(W) - u(G)]/u(W) \tag{8}$$

Furthermore, we can solve for $u(G)$ by using equation (8); thus,

$$u(G) = (1 - \Delta)\ u(W) \tag{9}$$

In comparing judgments of minimum selling price with the basic evaluation of a gamble (i.e., equation 7 with equation 9), note that the only difference is with respect to the subjective worth of the amount to win in the latter. Now consider the preference reversals involving minimum selling price judgments and choices (recall that the experimental results for these comparisons are shown in Table 2, column 1). We begin with the case where minimum selling price judgments for the P-Bet are lower than the $-Bet $(MS_p < MS_\$)$, but the P-Bet is chosen over the $-Bet $[u(G_p) > u(G_\$)]$. Using equation (7), the minimum selling price judgments imply that,

$$(1-\Delta_p)W_p < (1-\Delta_\$)W_\$ \tag{10}$$

Since the amount to be won in the $-Bet is much larger than in the P-Bet, this inequality is likely to hold even though $(1 - \Delta_p)$ is larger than $(1 - \Delta_\$)$. Now consider the choice of the P-Bet over the $-Bet. Using equation (9),

$$(1 - \Delta_p)\, u(W_p) > (1 - \Delta_\$)\, u(W_\$) \qquad (11)$$

Equations (10) and (11) show the typical reversal pattern in the comparison of minimum selling price judgments vs. choices. Note that it is the utility function that accounts for the change in the inequality since Δ_p and $\Delta_\$$ are the same in the two equations. The reversal occurs because the utilities of the amounts to win in the P-Bet and \$-Bet are closer together than the dollar amounts in the two bets. This implies that the utility function is concave, which is consistent with the fact that the P-Bet (almost a sure-thing), is chosen over the \$-Bet (which is "riskier").[5]

The above reversal is illustrated in Figure 3. The utility scale shows

Insert Figure 3 about here

that $u(G_p) > u(G_\$)$, and thus the P-bet is chosen. Moreover, it can be seen that Δ_p, the proportional adjustment down from $u(W_p)$, is smaller than $\Delta_\$$, the proportional adjustment down from $u(W_\$)$. However, in the subjective interpolation process, Δ_p becomes a proportional adjustment down from W_p, while $\Delta_\$$ becomes a proportional adjustment down from $W_\$$. Despite the fact that $\Delta_\$$ is larger than Δ_p, $W_\$$ sufficiently exceeds W_p so that $MS(G_\$) = (1-\Delta_\$)W_\$$ is larger than $MS(G_p) = (1-\Delta_p)W_p$. This yields the typical preference reversal. It should be emphasized that for any specific gamble, the mapping from $u(G)$ to $MS(G)$ is a monotonic function of $u(G)$. However, when considering two or more gambles together, the mapping from $u(G)$ to $MS(G)$ is not monotonic. A similar argument can be made for the comparison between choices versus ratings, and, selling prices versus ratings.

Now consider the other reversal pattern; viz., the minimum selling price is higher for the P-Bet, but the choice is for the \$-Bet. Using equations (7)

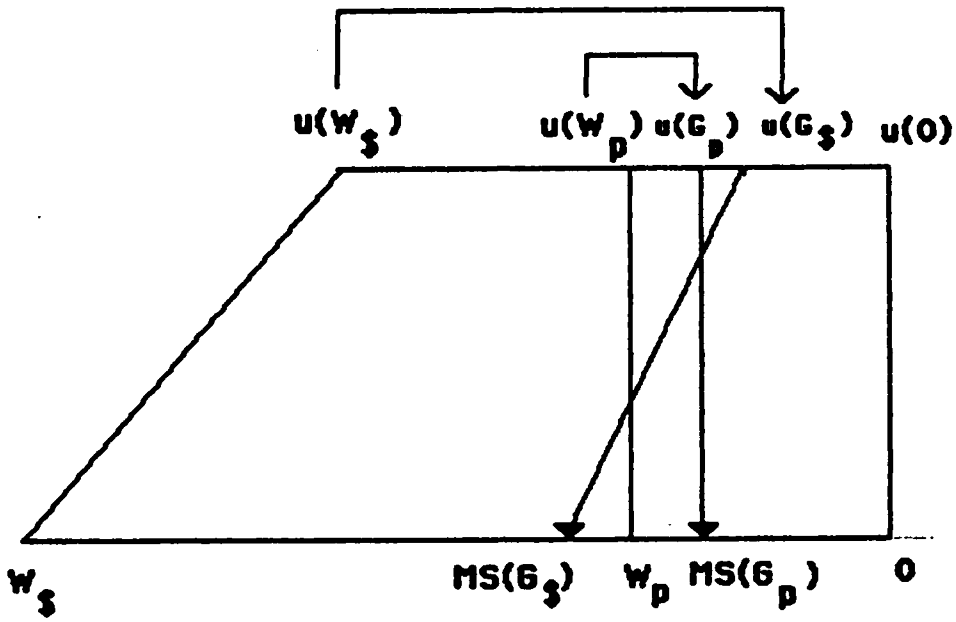

Figure 3. Subjective interpolation process in reversals of choice and minimum selling price.

and (9) again,

$$(1 - \Delta_p)W_p > (1 - \Delta_\$)W_\$$$

$$(1 - \Delta_p)\ u(W_p) < (1 - \Delta_\$)\ u(W_\$) \qquad (12)$$

Note that for the P-Bet to get a larger minimum selling price than the \$-Bet, $(1 - \Delta_p)$ must be considerably larger than $(1 - \Delta_\$)$. However, if this occurs, it makes the second inequality less likely to hold unless the utility function is convex. While this is possible, convex utilities for gains have not been found often in empirical studies. Therefore, our analysis shows that reversals can not only occur, but that the direction is much more likely for the case shown in equations (10) and (11) than in (12). This is the usual finding in the literature as well as in our data (see Table 2 above). We can summarize how our theory handles the case of minimum selling price judgments vs. choices, in Table 3. The table shows the 4 possible outcomes that result

Insert Table 3 about here

from ordering the P-Bet and \$-Bet and the theoretical inequalities that are implied from our model.

We now consider judgments of attractiveness and choices. Recall equation (6) and note that if the P-Bet gets a higher attractiveness rating than the \$-Bet, this implies that,

$$99(1 - \Delta_p) + 1 > 99(1 - \Delta_\$) + 1$$

Thus,

$$(1 - \Delta_p) > (1 - \Delta_\$) \qquad (13)$$

Equation (13) allows us to compare the attractiveness ratings to both choices and judgments of minimum selling price. Consider the comparison with minimum selling price judgments first; in particular, the reversal in which the P-Bet

TABLE 3

Theoretical Implications for Minimum Selling Price Judgments vs. Choices

		MS	
		P	$
Choice	P	$(1-\Delta_p)W_p > (1-\Delta_\$)W_\$$; $(1-\Delta_p)u(W_p) > (1-\Delta_\$)u(W_\$)$	$(1-\Delta_p)W_p < (1-\Delta_\$)W_\$$; $(1-\Delta_p)u(W_p) > (1-\Delta_\$)u(W_\$)$ (Likely)
	$	$(1-\Delta_p)W_p > (1-\Delta_\$)W_\$$; $(1-\Delta_p)u(W_p) < (1-\Delta_\$)u(W_\$)$ (Unlikely)	$(1-\Delta_p)W_p < (1-\Delta_\$)W_\$$; $(1-\Delta_p)u(W_p) < (1-\Delta_\$)u(W_\$)$

is rated higher, but the minimum selling price is lower than the \$-Bet. The former condition is shown in (13), and the latter condition is shown in (10); i.e.,

$$(1 - \Delta_p)W_p < (1 - \Delta_\$)W_\$$$

Since W_p is less than $W_\$$, (13) and (10) are not inconsistent. However, consider the case where the ratings for the \$-Bet are higher, but the P-Bet has a higher minimum selling price;

$$\begin{gathered}(1 - \Delta_p) < (1 - \Delta_\$) \quad \text{but,} \\ (1 - \Delta_p)W_p > (1 - \Delta_\$)W_\$\end{gathered} \tag{14}$$

Note that the two conditions in (14) are inconsistent, making this reversal impossible. In Appendix B, we show that in the case where losses are neither equal nor zero, and, $f(\Delta)$ and $g(\Delta)$ are not the identity function, it is possible but unlikely for such reversals to occur. Indeed, examination of Table 2 shows that only 3 responses out of 126 were of this type. Therefore, the theory accounts for the reversals between both types of judgments, including the very strong asymmetry found in this case. A summary of the theoretical inequalities that are implied by our model for the within-judgment comparisons are given in Table 4.

Insert Table 4 about here

Finally, the comparison of attractiveness ratings and choice reveals another impossible reversal; the judgment of attractiveness is higher for the \$-Bet, but the choice favors the P-Bet. That is,

$$\begin{gathered}(1 - \Delta_p) < (1 - \Delta_\$) \quad \text{but,} \\ (1 - \Delta_p)\, u(W_p) > (1 - \Delta_\$)\, u(W_\$)\end{gathered} \tag{15}$$

TABLE 4

Theoretical Implications for Attractiveness vs. Minimum Selling Price Judgments

		Attractiveness Ratings: P	Attractiveness Ratings: $
Minimum Selling Price	P	$(1-\Delta_p) > (1-\Delta_\$)$; $(1-\Delta_p)W_p > (1-\Delta_\$)W_\$$	$(1-\Delta_p) < (1-\Delta_\$)$; $(1-\Delta_p)W_p > (1-\Delta_\$)W_\$$ (Impossible)
	$	$(1-\Delta_p) > (1-\Delta_\$)$; $(1-\Delta_p)W_p < (1-\Delta_\$)W_\$$	$(1-\Delta_p) < (1-\Delta_\$)$; $(1-\Delta_p)W_p < (1-\Delta_\$)W_\$$

Since the utility function is monotonic with amount to win, (15) cannot occur because $u(W_P) < u(W_\$)$. In Appendix B we show that this reversal is possible with non-zero losses but is highly unlikely when the losses are similar. Again, the reader is invited to examine Table 2, column 2, to see how well our data fits this implication of the model. Finally, a summary of the implications of our model for the attractiveness ratings vs. choice situation is given in Table 5.

Insert Table 5 about here

RE-ANALYSIS OF TVERSKY-SLOVIC DATA

Expression Theory accords well with the data of our experiment. However, since it was developed with the results already known, any peculiarities in the data may have been capitalized on in constructing the theory. Thus, an independent data set is required for a "cross-validation" of the model. Such a data set, based on an experiment similar in design to ours, comes from a recent study by Tversky and Slovic (1984). We now describe the key differences between the two experiments.

First, the six pairs of gambles used by Tversky and Slovic can be formed from the gambles used in our experiment by deleting the potential losses. Thus, for example, the P-Bet in the first pair of gambles offered a 35/36 probability to win $4.00 and a 1/36 probability that no money would change hands. Second, Tversky and Slovic (1984) used three tasks: (1) attractiveness ratings on a scale from 0 to 20; (2) prices; and (3) choices based on attractiveness. Each subject performed two of the three tasks. Some subjects also performed an additional task in which they chose between each of the 12 gambles paired with a sure gain. Third, a manipulation was included to insure that there would be no strategic advantage to be gained by ordering the

TABLE 5

Theoretical Implications for Attractiveness Judgments vs. Choices

		Attractiveness Ratings	
		P	\$
Choice	P	$(1-\Delta_p) > (1-\Delta_\$)$; $(1-\Delta_p)u(W_p) > (1-\Delta_\$)u(W_\$)$	$(1-\Delta_p) < (1-\Delta_\$)$; $(1-\Delta_p)u(W_p) > (1-\Delta_\$)u(W_\$)$ (Impossible)
	\$	$(1-\Delta_p) > (1-\Delta_\$)$; $(1-\Delta_p)u(W_p) < (1-\Delta_\$)u(W_\$)$	$(1-\Delta_p) < (1-\Delta_\$)$; $(1-\Delta_p)u(W_p) < (1-\Delta_\$)u(W_\$)$

gambles differently in the three tasks. Subjects were told that a pair of bets would be selected and that they would play the bet that had been rated more highly (or priced more highly, or chosen).

In the previous section, we presented the predictions of Expression Theory for the special case of zero losses. This was done for simplicity of presentation. In the case of the Tversky and Slovic experiment, the losses were in fact equal to zero, and the predictions apply directly. Table 6 presents a summary of their results.

Insert Table 6 about here

First consider the comparison of prices and choices (Table 6a) Recall that Expression Theory permits both kinds of reversals, depending on the shape of the utility function. Reversals in which price favors the \$-Bet while choice favors the P-Bet involve people with concave utility functions. Only people whose utility functions are convex can have prices that favor the P-Bet while choice favors the \$-Bet. Since convex utility functions have seldom been observed empirically, we predict that the more likely reversal is one in which price favors the \$-Bet while choice favors the P-Bet. Note the results in Table 6a; the predicted reversal occurred 154 times out of 333 response patterns (46%) while the unpredicted reversal occurred only 21 times (6%). Thus, Expression Theory accounts for the predominant direction of reversals between minimum selling price and choice (the original preference reversal phenomenon).

Now consider the comparison of ratings and prices (Table 6b). Recall that Expression Theory predicts that one of the two reversals will <u>never</u> occur. The model implies that it is impossible for subjects to set a higher price for the P-Bet but rate the \$-Bet more highly. The permissible reversal consists of setting a higher price for the \$-Bet while rating the P-Bet more

TABLE 6

Tversky-Slovic Data

(a)

Choice \ Price	P	$	
P	62	154	216
$	21	96	117
	83	250	333

(b)

Rating \ Price	P	$	
P	21	128	149
$	2	23	25
	23	151	174

(c)

Choice \ Rating	P	$	
P	331	12	343
$	146	47	193
	477	59	536

highly. In Table 6b it is seen that the "impossible" reversal occurred exactly twice out of 174 response patterns (1%), while the "possible" reversal occurred 128 times (74%). Thus, Expression Theory captures the asymmetry in the relative frequencies of the two reversals.

Finally, consider the comparison of ratings and choices (Table 6c). Recall that Expression Theory predicts one of the two kinds of reversals to be impossible. If the model is correct, one cannot choose the P-Bet but give a higher rating for the $-Bet. However, it is possible to choose the $-Bet while rating the P-Bet more highly. Table 6c shows that the "impossible" reversal occurred 12 times out of 536 response patterns (2%), while the "possible" reversal occurred 146 times (27%). Again, the "impossible" reversal occurred rarely, while the "possible" (and frequent) reversal is easily interpretable within Expression Theory.

Overall, the same pattern of results is exhibited in our experiment and in the Tversky-Slovic experiment. The same reversals in the same directions are found in the two experiments. Moreover, the method in which the Tversky-Slovic experiment was conducted allows us to reject two alternative explanations for the data of the experiment reported here. First, since gambles were actually played for real payoffs, we can reject the hypothesis that reversals are due to insufficiently motivated subjects responding in a thoughtless manner. Second, the way Tversky and Slovic operationalized the three tasks eliminates the possibility that there was some strategic (i.e., normative) discrepancy among the tasks. Finally, the fact that reversals remained in Tversky and Slovic's data despite their use of simpler gambles than previously used (potential losses were eliminated), speaks to the robustness of the phenomenon.

<u>Expression theory and the compatibility hypothesis</u>. We have reanalyzed the Tversky and Slovic data in order to test predictions of Expression Theory. In our view, the theory provides a good account of the data. However, the data are also explained by a model proposed by Tversky and Slovic (1984). In this section, we first present the predictions from their "differential weighting model" and then compare the two approaches.

Tversky and Slovic begin by proposing a compatibility hypothesis, which states that, ". . . the easier the mapping of a stimulus component onto the response scale, the greater the weight attached to this component. Because pricing is expressed in monetary units, a gamble's monetary payoffs may be weighted relatively heavily. In contrast, because choice and ratings of attractiveness are not expressed in monetary units, they should be less sensitive than pricing to the gamble's payoffs" (p. 6). They then use the compatibility hypothesis as a rationale for developing a differential weighting model, which they state algebraically and axiomatize. For the present purposes, we can summarize the differential weighting model pictorially.

Consider Figure 4, which shows a two dimensional space comprised of the amount to win on the abscissa, and the probability of winning on the ordinate. Any point in this space represents a gamble with zero loss, where the zero loss occurs with a probability that is complementary to that shown on the

<u>Insert Figure 4 about here</u>

ordinate. Note that P-Bets would tend to be found in the upper left part of the space (small amount to win and high probability of winning), while \$-Bets would tend to be found in the lower right part (large amount to win and small probability of winning).

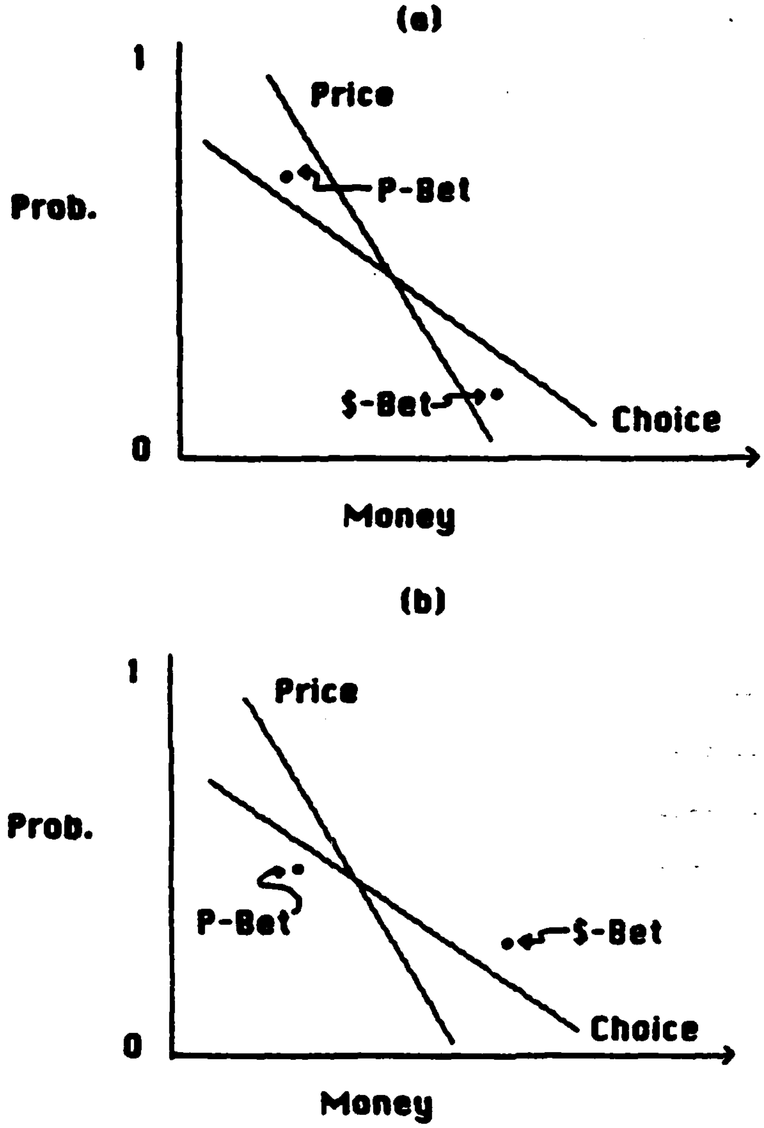

Figure 4. Differential weighting model predictions for the choice vs. price comparison.

Consider Figure 4a first, which shows two indifference curves in this space. These curves are shown as linear for illustrative purposes but this is not an essential feature of the differential weighting model. A P-Bet and $-Bet are also shown in the Figure. For the moment, only consider the position of the P-Bet and $-Bet with respect to the indifference curve labeled "choice" (this is the less steep line). Note that the P-Bet is to the right of the indifference line while the $-Bet is to its left. This means that the P-Bet lies on a higher indifference curve than the $-Bet and should thus be chosen over it. The critical feature of the differential weighting model is that the slopes of the indifference curves for probability and payoffs depend on the task the subject is performing. In particular, the slope is steepest for the pricing task, since the relative weight for money is greater than for probability (due to the compatibility of money with the pricing task). This indifference function is also shown in Figure 4a and is labeled, "price." Note the position of the P-Bet and $-Bet with respect to this indifference curve; the P-Bet is now to the left of the curve while the $-Bet is to the right. This means that the $-Bet will have a higher price than the P-Bet even though the P-Bet is chosen. Hence, the differential weighting of money in the pricing task leads to a reversal between choices (the P-Bet is chosen) and prices (the $-Bet gets the higher price).

While the differential weighting model can thus account for the usual reversal found in the choice vs. price situation, consider Figure 4b, which shows the P-Bet to the left of the choice indifference curve and the $-Bet to its right. This implies that the $-Bet will be chosen over the P-Bet. However, imagine any indifference curve for the price task, which has a steeper slope. Note that regardless of the position of the price indifference curve, the $-Bet will always be to the right of the P-Bet. This means that the $-Bet

must have a higher price than the P-Bet if the $-Bet is chosen. Therefore, the differential weighting models implies that the reversal in which the $-Bet is chosen but the P-Bet gets a higher price, is impossible. We can contrast this implication with Expression Theory, where this reversal is possible but unlikely (see Table 3). If we consider the data from the Tversky and Slovic experiment relevant to this comparison (shown in Table 6a), we find that 21 out of 333 responses (6%) show the reversal. Whether or not this percentage reflects error in the data or a violation of the model, cannot be determined.

Now consider Figure 5, which shows the indifference curves for the price vs. rating comparison. Tversky and Slovic suggest that the rating task is one in which probability will receive more weight than in the price task. In Figure 5a, one can see that it is possible for the $-Bet to get a higher price

Insert Figure 5 about here

and a lower rating. However, Figure 5b shows that a reversal in which the P-Bet gets a higher price and a lower rating is impossible. This prediction from the differential weighting model is identical to our prediction from Expression Theory (see Table 4). The data from Table 6b are quite consistent with this prediction (2 responses out of 174, or 1%).

The third comparison, between choice and rating, can be analyzed in the same way. To save space, we simply present the results: the differential weighting model and Expression Theory both predict that the reversal in which the P-Bet is chosen and the $-Bet gets a higher rating, is impossible. The data from Table 6c show that only 12 of 536 responses (2%) were of this type.

We now consider a general discussion of the preference reversal phenomenon, including further analysis of the differences between Expression Theory and the differential weighting model of Tversky and Slovic (1984).

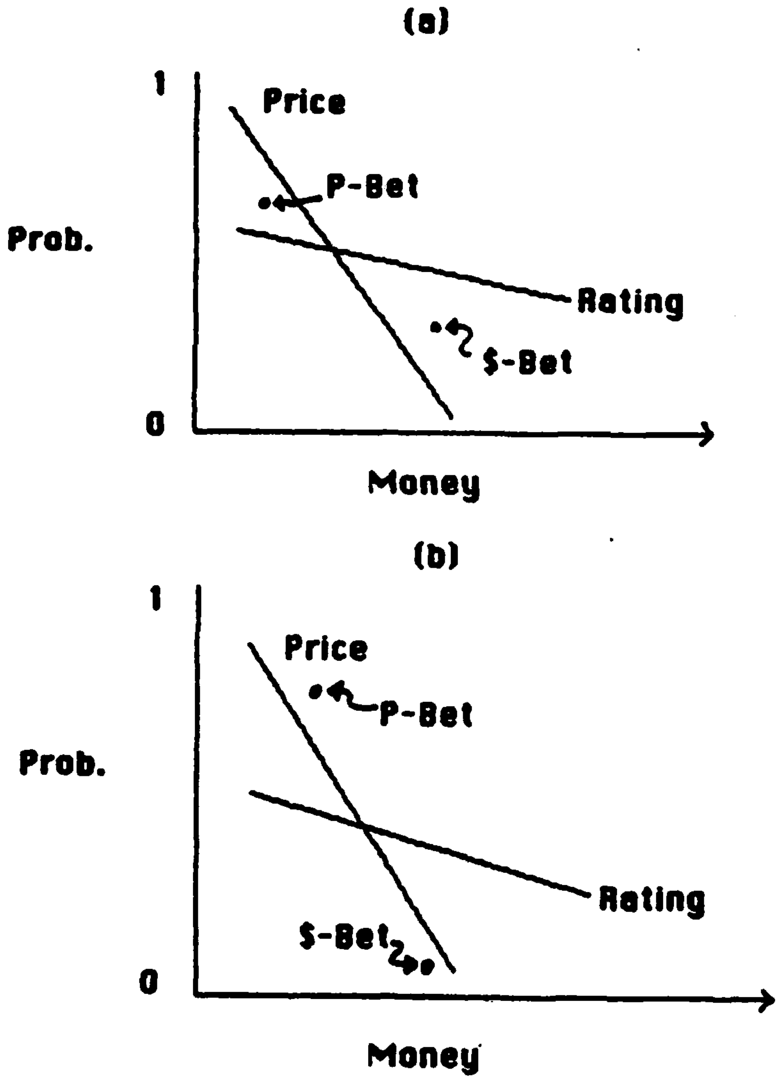

Figure 5. Differential weighting model predictions for the price vs. rating comparison.

GENERAL DISCUSSION

Although the preference reversal phenomenon was first demonstrated in 1971 by Lichtenstein and Slovic (and its possibility suggested earlier by Slovic & Lichtenstein, 1968), there has been remarkably little theoretical work concerned with it in the intervening years. To be sure, hypotheses have been advanced and experimentally tested (see the introduction), but a quantitative model has been missing. Our theory has the merit of hypothesizing a simple model based on subjective interpolation and anchoring-and-adjustment that not only explains the original reversal phenomenon, but several others as well. Moreover, the theory is eminently falsifiable in that it makes strong predictions about the impossibility of certain reversals under specified conditions. Therefore, given its parsimonious explanation of the phenomenon, and its excellent fit to the data, Expression Theory deserves further discussion. To do so, we first consider the theory with respect to its psychological assumptions. Thereafter, its generality is discussed with respect to the following questions: is the theory consistent with the fact that negative gambles show reversals that are the reverse of positive gambles?; can the theory explain violations of dominance that have recently been observed (Slovic, personal communication)?; can the theory be used to predict actual prices and ratings in addition to orderings?; and, to what other types of worth scales and gambles can the theory be applied? Finally, we compare Expression Theory with the differential weighting model of Tversky and Slovic (1984).

Psychological assumptions

In formulating our model, we have assumed that the evaluation of gambles on the basis of probabilities and payoffs occurs prior to the expression of that evaluation in an overt response. Furthermore, although we believe that

the basic evaluation process involves an anchoring-and-adjustment process based on the utilities of the gamble, we have provided no evidence for this assumption. However, Expression Theory is only minimally dependent on specifying how the basic evaluation is accomplished. Nevertheless, we feel that sufficient evidence exists to independently support the idea of anchoring-and-adjustment. For example, Tversky and Slovic (1984) report that when subjects were given different arbitrary anchors for evaluating gambles, they gave different responses. Moreover, much other work in judgment and choice is consistent with anchoring-and-adjustment strategies (Lopes, 1981; Einhorn & Hogarth, 1984).

The major assumption of our model concerns the monotonic relation between the proportional change in the utility scale (Δ) and the proportional changes in the worth scales (Δ' and Δ'' - see equations (3) and (5)). We have likened this process to a "subjective interpolation," since one must find a point on the worth scale between W and L, or between high and low points on the rating scale, that corresponds to the basic evaluation on the utility scale. By equating the end points of the utility scale to points on the worth scales, a translation of responses from one to the other can be made more easily. The strongest evidence for the subjective interpolation process comes from the reversals involving the ratings of attractiveness. Recall that subjects were asked to rate, on a scale from 1-100, how much they would like to play a particular gamble. Examination of the <u>level</u> of these ratings revealed that the P-Bets were rated an average of 79, while the $-Bets had an average rating of 58.5. This large difference is consistent with the fact that the reversals involving ratings were the strongest in both our study and the Tversky-Slovic experiment. Indeed, the average ratings in their study were quite similar to ours despite the fact that their gambles had zero

losses; 16.5 and 10.4 for the P-Bets and \$-Bets, respectively (on a 20-point scale).

How is the difference in ratings accounted for by the subjective interpolation process? Consider a P-Bet in which you win \$4 with $p = .97$ and lose \$1 otherwise. Since this is almost a "sure-thing," there is a small proportional adjustment in the basic evaluation and the gamble is rated as very attractive (an average of 92 in our study; an average of 18.9 out of 20 in the Tversky and Slovic study). Note that such a high rating leaves little room at the top of the rating scale for gambles such as, win \$40 with $p = .97$ and lose \$1 otherwise (clearly a better bet than the first). If subjects are equating the utility of the amount to win, $u(W)$, with a high point on the rating scale, and $u(L)$ with some low point, a small proportionate adjustment in utility will be translated into a high rating. This explanation suggests that the degree to which people are aware of being offered even better (or worse) gambles, will affect the amount of space they leave from the end points of the scale. In these experiments, subjects take a limited perspective. Now consider the rating for a \$-Bet in which you win \$16 with $p = .31$ and lose \$1.50 otherwise. If $u(\$16)$ is equated with a high point on the rating scale and $u(-\$1.50)$ with some low point, then a large proportional adjustment in the basic evaluation will translate into a low rating (an average of 58 in our study; an average of 11 in the Tversky and Slovic study). Therefore, the notion that subjects use some type of subjective interpolation process is consistent with the substantial discrepancies in the level of ratings for the P and \$ bets. We consider the issue of predicting such ratings (and minimum selling prices) later.

Generality of expression theory

Reversing reversals. In an experiment in a Las Vegas casino, Lichtenstein and Slovic (1973) collected prices and choices for an unusual set of gambles. These gambles were derived from the more usual P and $ bets by multiplying all outcomes by -1. They found that the predominant direction of reversals reversed; i.e., the $-Bet was now chosen more often than the P-Bet although the latter received the higher price. Note that this is not as peculiar as it might seem since the P-Bet is now an almost sure loss, while the $-Bet gives one a reasonable chance of a small win (at the risk of a big loss). Expression Theory can accommodate such results by assuming that the basic evaluation of the $-Bet is higher than the P-Bet, and, the proportional adjustment is greater for the $-Bet [i.e., $\Delta_\$ > \Delta_P$, which results in lower prices for the $-Bet; see equation (12)]. In considering losses, the choice of the $-Bet over the P-Bet (as well as the above reversal) is consistent with a convex utility function for losses that has been posited by Kahneman and Tversky (1979). Indeed, a convex utility function for losses results in what they call "reflection effects," wherein choices for losses are mirror-images of choices for gains. From the point of view of reversals between choices, ratings, and prices, "reflection effects" lead to reversing reversals.

Violations of dominance. A basic tenet of all choice theories is that choices should preserve dominance; i.e., given two gambles that are otherwise the same, one should choose the one with the larger amount or probability to win (or, the smaller amount or probability to lose). While violations of various principles of rational choice have been found in the behavioral literature, violations of dominance have rarely been noted. However, Slovic (personal communication) has reported the following study in which dominance was consistently violated. Subjects were given a gamble in which they could

win \$9 with probability 7/36 and \$0 otherwise. They were then asked to rate the attractiveness of playing this bet on a 0-20 scale. The average rating (over 60 subjects) was 9.4. Two other groups (of approximately 60 each) were given the identical bet except that the zero loss was changed to a loss of 5 and 25 cents, respectively. The average ratings for these two groups were 14.9 (5 cents loss), and 11.7 (25 cents loss). Thus, by increasing the amount of the loss in the gamble, Slovic was able to get subjects to rate it as more attractive! Can Expression Theory handle such results? In order to investigate this, let G_0 denote the gamble in which there is a zero loss, and G_1 the gamble in which the loss is 5 cents. We will assume that dominance is preserved with respect to the basic evaluation; i.e., $u(G_0) > u(G_1)$. However, in translating the basic evaluation onto the rating scale, if the proportional adjustment in utility for G_0 is greater than for G_1 (i.e., $\Delta_0 > \Delta_1$), the rating for G_0 will be lower than for G_1 and dominance will be violated <u>on the rating scale</u>. Is this possible according to our model? That is, can $\Delta_0 > \Delta_1$ if $u(G_0) > u(G_1)$? To examine this, we can express Δ as a function of the basic evaluation by using equation (2); thus,

$$\Delta_0 = \frac{u(\$9) - u(G_0)}{u(\$9) - u(\$0)} \quad \text{and} \quad \Delta_1 = \frac{u(\$9) - u(G_1)}{u(\$9) - u(-.05)} \qquad (16)$$

If $u(G_0) > u(G_1)$, the numerator for Δ_0 is smaller than for Δ_1. However, note that the denominator for Δ_0 is also smaller than for Δ_1 (since $u(0) > u(-.05)$). Therefore, it is possible for $\Delta_0 > \Delta_1$ and thus, for violations of dominance to occur in our model. Note that such violations can only happen on the various worth scales and not in the basic evaluations themselves. Violations of the latter type cannot be handled by Expression Theory.

Figure 6 illustrates the violation of dominance described above. In

Insert Figure 6 about here

accordance with dominance on the utility scale, the figure shows that $u(G_0) > u(G_1)$. Of course, the absolute adjustment down from $u(\$9)$ is less for G_0 than for G_1, i.e., $u(\$9) - u(G_0) < u(\$9) - u(G_1)$. However, the proportional adjustment down from $u(\$9)$ is greater for G_0 than for G_1, because the denominator of the proportion is sufficiently greater for G_1. That is,

$$\Delta_0 = \frac{u(\$9) - u(G_0)}{u(\$9) - u(\$0)} > \frac{u(\$9) - u(G_1)}{u(\$9) - u(-.05)} = \Delta_1$$

In the subjective interpolation process, these adjustments become proportional adjustments down from a maximum rating of 20. Thus, G_1 receives a higher rating than G_0, a violation of dominance on the rating scale.

Numerical predictions of ratings and prices. In its current form, Expression Theory concerns the ordering of prices and ratings, not their numerical values. However, recall that both worth scales are monotonic with the proportional adjustment in utility, Δ. That is, from equations (4) and (6), we can write,

$$\begin{aligned} f(\Delta) &= [W - MS(G)]/[W - L) \quad \text{and,} \\ g(\Delta) &= [100 - R(G)]/(100 - 1) \end{aligned} \tag{17}$$

Since f and g are both monotonic functions of Δ, they are monotonic with each other. Thus, Expression Theory predicts that proportional adjustments in prices will be monotonic with proportional adjustments in ratings.

A numerical relationship between prices and ratings can be predicted if

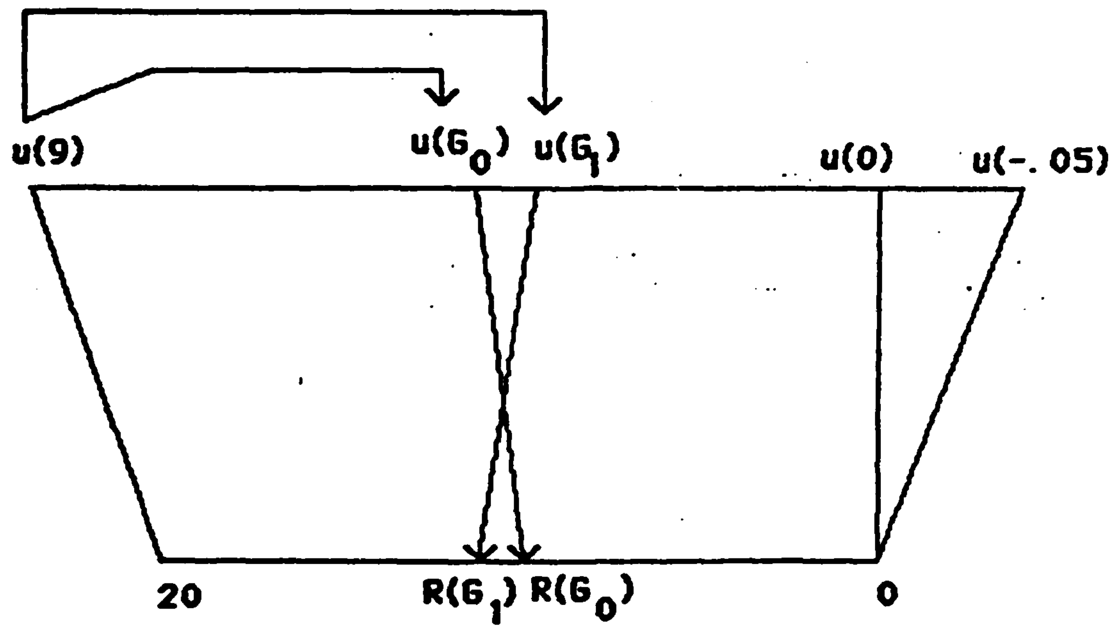

Figure 6. Subjective interpolation process in violation of dominance.

one is willing to assume functional forms for f and g. For example, suppose that $f(\Delta) = \Delta^{\alpha}$ and $g(\Delta) = \Delta^{\beta}$. By using (17), it can be shown that,

$$MS(G) = W - (W - L)\ \{[100 - R(G)]/99\}^{\alpha/\beta},$$

and, (18)

$$R(G) = 100 - 99\{[W - MS(G)]/(W - L)\}^{\beta/\alpha}$$

Thus, after fitting the parameter α/β, prices can be predicted from ratings, or vice versa. It should be noted that until this point, Expression Theory has not required the estimation of any parameters.

Other assessment methods. Because the original preference reversal phenomenon involved the response methods of judgment and choice and the worth scales of minimum selling price and attractiveness, we have not considered other methods of assessment. However, another worth scale of considerable practical importance is maximum buying price. While judgments of this variable have been included in some studies of the preference reversal phenomenon (Lichtenstein & Slovic, 1971; Hamm, 1979), it has not received the same scrutiny as minimum selling price because expected utility theory permits maximum buying prices to be ordered differently from choices and minimum selling prices (Raiffa, 1968). In fact, there is empirical evidence that gambles are ordered differently by maximum buying price and minimum selling price (Goldstein, unpublished data). Expression Theory could include maximum buying prices by positing the existence of a third function h, which is monotonic with Δ, such that

$$MB(G) = W - h(\Delta)[W - L] \tag{19}$$

If one were to assume a specific form for h, it would be possible to predict maximum buying prices from ratings and minimum selling prices.

Another assessment method receiving considerable attention has been

called "probability equivalence" (Johnson and Schkade, 1984). In this method, a subject is shown a sure gain S, and a gamble with the payoffs W and L. However, the probability of winning W is not shown. The subject's task is to judge the probability p, which would make him or her indifferent between the gamble (W, p; L, 1-p) and the sure gain S. The method of probability equivalence has been shown to be discrepant from the analogous method of "certainty equivalence" (sometimes operationalized as a minimum selling price judgment) (Hershey, Kunreuther, and Schoemaker, 1982; Hershey and Schoemaker, 1983; Johnson and Schkade, 1984). While Expression Theory has not been applied to these assessment methods so far, we believe it has much potential for doing so.

Comparison with the differential weighting model. Expression Theory and the differential weighting model make very similar predictions for the data at hand. Indeed, only one prediction is different for the two models; in the case where the \$-Bet is chosen over the P-Bet, but the P-Bet gets the higher price. We predict that such a reversal is possible but unlikely the differential weighting model predicts that this reversal is impossible. The data from the Tversky-Slovic study shows that this reversal occurred for 6% of the responses, significantly higher than the 1% and 2% for reversals that both models imply are impossible ($\chi^2 = 7.02$, $p < .01$ and, $\chi^2 = 9.3$, $p < .01$, respectively). Thus, Expression Theory seems to fit the data somewhat better than the differential weighting model. Of greater importance is the fact that Expression Theory is consistent with ratings that violate dominance, while the differential weighting model is not. Indeed, if consistent violations of dominance prove to be a stable and pervasive phenomenon, this would provide compelling evidence for our approach. Finally, Expression Theory can be extended to the numerical prediction of ratings and prices as well as to other worth scales. The differential weighting model, on the other hand, seems more

closely tied to orderings.

Thus far, we have only compared the two models on output measures. However, because the two theories rest on very different assumptions about the processes underlying the reversal phenomenon, each focuses attention on somewhat different issues that can be further studied. For example, consider the difference between judgment and choice in Expression Theory. To form a numerical judgment, one first makes a basic evaluation of the gamble and then translates it on to a worth scale. In choice, one compares both gambles on their basic evaluations. Which kind of response method is faster? Note that choice involves comparing basic evaluations on the same scale, while judgment involves an inter-scale comparison. Since research indicates that intra-attribute comparisons are easier than between-attribute comparisons (see, e.g., Russo & Dosher, 1983), we predict that choices will generally be made faster than judgments. Since the differential weighting model only speaks to the final ordering, these sorts of questions do not arise. By contrast, the differential weighting model addresses questions that are not easily handled by Expression Theory. For example, the role of attentional factors in making the dimensions of gambles more salient (thereby increasing their weight), is easily accommodated. Indeed, the model has already been used to explain phenomena in which subjects weight certain types of information more heavily than others (Slovic & McPhillamy, 1974). Therefore, we see considerable complementarity of the two approaches. Indeed, there is no reason to believe that any single explanation will be sufficient for explaining preference reversals in all their complexity.

CONCLUSION

We began this paper by pointing out that the preference reversal phenomenon poses an important challenge to theories of judgment and choice. That is,

the idea that different worth scales can lead to reversals in the order of preference raises the disturbing possibility that theories of choice and judgment must be conditional on the specific ways in which preferences are assessed. Since there are innumerable response scales, the possibility of achieving generality in theories of decision making would seem to be seriously threatened. Note, however, that the difficulty of achieving generality is not peculiar to decision research; indeed, the problem of "method variance" (Campbell & Fiske, 1959) plagues all science (and social science in particular). For example, it is well known that the form of the question can greatly affect responses in a wide variety of contexts (see Hogarth, 1982). Therefore, preference reversals are an extreme example of the more general problem concerning the sensitivity of responses to the way they are elicited.

Our approach to understanding preference reversals has been to posit a simple psychological process that is general, yet captures the complexity of responses that have been found in the literature. To do this, we have proposed a model whereby people translate their basic evaluations of gambles onto various worth scales by using a proportional adjustment matching strategy. While we believe that our model provides a testable, generalizable, and parsimonious explanation of the phenomenon, its real importance lies in showing that the generality of theory can be achieved, even in the face of seemingly inconsistent and contradictory responses.

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Appendix A

A strategy for axiomatizing Expression Theory is sketched below. Let A be a set of monetary values, and let G be the set of two-outcome gambles involving outcomes in A. Let $(x,p;y,1-p) > (z,q;w,1-q)$ indicate that the gamble $(x,p;y,1-p)$ is chosen over $(z,q;w,1-q)$. Let $MS(x,p;y,1-p)$ and $R(x,p;y,1-p)$ denote the minimum selling price and attractiveness rating, respectively, for the gamble $(x,p;y,1-p)$. The goal is to find conditions on the structure $(G,MS,R, >)$ which are sufficient to insure the existence of functions $u{:}A \rightarrow Re$, $\Delta\ G \rightarrow [0,1]$, and strictly increasing functions f and g from $[0,1]$ into $[0,1]$ such that for any $(x,p;y,1-p)$, $(z,q;w,1-q)$ G,

$$(x,p;y,1-p) > (z,q;w,1-q)$$

iff

$$u[\max(x,y)][1-\Delta(x,p;y,1-p)] + u[\min(x,y)]\Delta(x,p;y,1-p)$$

$$> u[\max(z,w)][1-\Delta(z,q;w,1-q)] + u[\min(z,w)]\Delta(z,q;w,1-q),$$

$$MS(x,p;y,1-p) = \max(x,y)\{1-f[\Delta(x,p;y,1-p)]\}$$

$$+ \min(x,y)f[\Delta(x,p;y,1-p)],$$

and for any nondegenerate $(x,p;y,1-p)$ G,

$$R(x,p;y,1-p) = 100\{1-g[\Delta(x,p;y,1-p)]\} + g[\Delta(x,p;y,1-p)].$$

We can axiomatize the choice relation by following a strategy similar to that used by Ramsey (1931) to axiomatize subjective expected utility theory. First, we find a group of gambles for all of which $\Delta = .5$. Then we impose the axioms of additive conjoint measurement on the gambles with $\Delta = .5$, obtaining a utility function defined on the payoffs. For the remaining gambles, we solve for the Δ's in terms of the utilities. Finally an additional two axioms enable us to express MS and R as desired.

Since f is to be strictly increasing, it follows from the desired representation that $f[\Delta(x,p;y,1-p)] = f[\Delta(z,q;w,1-q)]$ iff $\Delta(x,p;y,1-p) = \Delta(z,q;w,1-q)$. This gives us a way of identifying gambles which must give rise to the same value of Δ. Let the set of gambles for which $\Delta'(x,p;y,1-p) = [\max(x,y) - MS(x,p;y,1-p)]/[\max(x,y) - \min(x,y)] = r$ be denoted G_r. All of the gambles G_r must have the same value of Δ, but we don't know what it is. Let the set E_r "enlarge" upon G_r by defining $E_r = G_r \quad \{(x,.5,x) \mid x \quad A\}$. For $x,y \quad A$, if there exists p such that $(x,p;y,1-p) \quad E_r$, it will be convenient to denote the gamble $(x,p;y,1-p)$ by $(x,y)_r$.

The key step is to identify that particular set of gambles G_{r*} for which Δ is equal to .5. To do this, the following axiom is imposed.

<u>Axiom 1</u>. There exists a set G_{r*} containing gambles $(y,p;z',1-p)$, $(y',p';z,1-p')$, $(x,q;z',1-q)$, $(x',q';z,1-q')$, $(x,r;y',1-r)$, and $(x',r';y,1-r')$, where $x>x'>y>y'>z>z'$, such that

$$(y,p;z',1-p) \sim (y',p';z,1-p'),$$
$$(x,q;z',1-q) \sim (x',q';z,1-q'),$$
$$\text{and } (x,r;y',1-r) \sim (x',r';y,1-r').$$

It is not hard to show that a necessary consequence of the desired representation is that all of the gambles in the set G_{r*} have $\Delta = .5$. Thus, for the gambles in G_{r*}, the desired representation reduces to $(x,y)_{r*} \succsim (z,w)_{r*}$ iff $u(x) + u(y) \geq u(z) + u(w)$. Axioms for this sort of additive structure are well-known (Krantz, Luce, Suppes, and Tversky, 1971). We impose conditions to guarantee that for any $(x,y) \quad A \times A$, there is a

gamble $(x,p;y,1-p)$ in E_{r*}, and we impose conditions for additivity on this set. This gives us a real-valued function u defined on A which is unique up to a positive affine transformation. Then for any gamble $(x,p;y,1-p)$ with $x \neq y$, we solve for Δ as follows. First we find the degenerate gamble $(z,.5,z)$ such that $(z,.5,z) \sim (x,p;y,1-p)$. Then setting $u(z) = u[\max(x,y)][1-\Delta(x,p;y,1-p)] + u[\min(x,y)]\Delta(x,p;y,1-p)$, we compute

$$\Delta(x,p;y,1-p) = \frac{u[\max(x,y)] - u(z)}{u[\max(x,y)] - u[\min(x,y)]}.$$

For degenerate gambles of the form (z,p,z), we set $\Delta=0$.

Finally, it can be shown that the desired expressions for minimum selling prices and attractiveness ratings can be derived if the following two axioms are satisfied.

<u>Axiom 2</u>. For any $(x,p;y,1-p)$, $(z,q;w,1-q)$ G, $\Delta(x,p;y,1-p) > \Delta(z,q;w,1-q)$ iff $\Delta'(x,p;y,1-p) > \Delta'(z,q;w,1-q)$.

<u>Axiom 3</u>. For any nondegenerate $(x,p;y,1-p)$, $(z,q;w,1-q)$ G, $\Delta(x,p;y,1-p) > \Delta(z,q;w,1-q)$ iff $R(x,p;y,1-p) < R(z,q;w,1-q)$.

Appendix B

We wish to show that the prediction of impossible reversals, which occurs when losses are zero and $f(\Delta)$ and $g(\Delta)$ are set equal to Δ. are highly unlikely when these restrictions are relaxed. To do so, let,

$$a = 1 - f(\Delta_\$) \text{ and, } b = 1 - f(\Delta_p) \tag{B.1}$$

$$a' = 1 - \Delta_\$ \text{ and, } b' = 1 - \Delta_p \tag{B.2}$$

$$c = 1 - g(\Delta_\$) \text{ and, } d = 1 - g(\Delta_p) \tag{B.3}$$

Consider the impossible reversal involving minimum selling prices vs. ratings; i.e., $MS_p > MS_\$$ and $R_p < R_\$$. Substituting equations (4) and (B.1) into the selling price inequality yields,

$$b\, W_p + (1-b)L_p > a\, W_\$ + (1-a)L_\$ \tag{B.4}$$

Similarly, using equations (6) and (B.3) for ratings yields,

$$d < c \tag{B.5}$$

From (B.4), note that since $W_\$ > W_p$, either $b > a$, or, $L_p > L_\$$ (or both). If $b > a$, note that $d > c$ since f and g are monotone with each other. In the case where losses are both zero, $b > a$ for (B.4) to hold. However, $d > c$ violates (B.5), and thus, $R_p < R_\$$ is impossible. However, if L_p is a larger amount than $L_\$$, then (B.4) and (B.5) can both hold (depending on the sizes of a,b,c, and d). In our experiment, and in all of the studies reviewed above, L_p and $L_\$$ have not been very discrepant. Therefore, while it is possible for (B.4) and (B.5) to hold in our experiment, it is quite unlikely.

Now consider the impossible reversal involving ratings and choices; i.e., $R_p < R_\$$ and $u(G_p) > u(G_\$)$. For ratings, $R_p < R_\$$ implies that

$d < c$ (as in (B.5)). The choice of the P-Bet over the \$-Bet implies,

$$b'u(W_p) + (1-b')u(L_p) > a'u(W_\$) + (1-a')u(L_\$) \qquad (B.6)$$

Note that if $d < c$, $b' < a'$ (via monotonicity). Moreover, since $u(W_p) < u(W_\$)$, equation (B.6) can hold if $u(L_\$)$ is considerably less than $u(L_p)$. This means that there would have to be a substantial discrepancy in losses for the P and \$ bets (or a very steep slope in the utility function containing L_p and $L_\$$) for this reversal to occur. Again, while this is possible, it is unlikely given the stimuli in this experiment.

FOOTNOTES

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[1]In studies of the preference reversal phenomenon, subjects have usually judged the monetary value of options by giving minimum selling prices. However, other kinds of judgments have occasionally been used. As well as minimum selling price, Grether and Plott (1979) elicited judgments of monetary value of gambles by a method which avoids references to market-type behavior. Grether and Plott asked subjects to give "the exact dollar amount such that you are indifferent between the bet and the amount of money" (Grether & Plott, 1979, p. 631). Maximum buying prices were included in studies by Lichtenstein and Slovic (1971, Experiment 1) and by Hamm (1979).

[2]In an experiment reported by Lindman (1971), subjects reported their minimum selling prices while the gambles were all simultaneously displayed. The intent was to permit subjects to make comparisons across the gambles while setting their prices. When subjects make choices with respect to minimum selling price, as in the lower left cell of Figure 1, some degree

of comparison is required.

[3]Smith's as yet unpublished work is related to work by Rao and Scott (1981, 1984).

[4]For convenience, we define the proportional adjustment in the ratings, Δ'', to be based on the entire length of the rating scale, i.e., $\Delta'' = [100 - R(G)]/[100 - 1]$. Only minor changes would be required if we were to assume that subjects reserve some room at the top and bottom of the rating scale for extraordinary gambles. In this case, the subjective interpolation process would be one in which the person first establishes a correspondence between the endpoints of the utility scale and some nearly extreme ratings, say 90 and 10. Then, he/she would attempt to find a rating such that the proportional adjustment in basic evaluation, Δ, matches the proportional adjustment in ratings, in this case defined as $\Delta'' = [90 - R(G)]/[90 - 10]$.

[5]From Equations *(10)* and *(11)*, we have $\frac{u(W_\$)}{u(W_p)} < \frac{1-\Delta_p}{1-\Delta_\$} < \frac{W_\$}{W_p}$. From this inequality, and the assumption that $u(0) = 0$, we can write $\frac{u(W_\$) - u(0)}{W_\$ - 0} < \frac{(W_p) - u(0)}{W_p - 0}$. Thus, the slope of the line from the origin through $[W_p, u(W_p)]$ is steeper than the slope of the line from the origin through the point $[W_\$, u(W_\$)]$. Since $W_p < W_\$$, a "smooth" utility function with this property is convex.

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